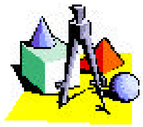


Middle School Geometry
Session 1

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
van Hiele Theory of Geometric Thought	Lecture – van Hiele Levels of Geometric Thought Triangle Sorts	4	6.14, 6.15	Explanation Sheets -van Hiele levels, -Additional Points, Triangle Sorting Pieces, Pages 1-4 Sample Student Sorts 1-6	Paper triangles
Quadrilaterals and Their Properties	Quadrilateral Sort	20	6.14, 7.9	Quadrilateral Sorting Pieces	Paper quadrilaterals
	What's My Rule?	23	6.14, 7.9	What's My Rule?	Paper quadrilaterals
	Quadrilateral Properties Laboratory	25	6.14, 7.9	Types of Quadrilaterals	Geo-strips, D-stix, or miniature marshmallows and toothpicks; square corner
	Quadrilateral Sorting Laboratory	29	6.14, 7.9	Quadrilateral Sorting Laboratory, Quadrilateral Table, Quadrilateral Family Tree	Paper quadrilaterals
Plane Figures and Their Properties	Polygons and the Geoboard	35	6.14, 6.15, 7.9, 7.10		Geoboards, rubber bands
	A Country Mile	36	6.11, 6.14, 7.7, 7.9, 7.10	A Country Mile Writing Prompt	Geoboards, rubber bands

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
	Similarity and Congruence with Geostrips	38	6.14, 6.15, 7.9, 7.11		Geostrips, protractors, or angle rulers
	Perimeters and Areas of Similar Triangles	42	6.11, 7.7, 7.11	Data Chart: Area and Perimeter of Similar Triangles	Dynamic geometry software program
	Making and Using a Hypsometer	47	6.15, 7.11		Straws, decimal graph paper, cardboard, thread, a small weight, tape, hole punch, scissors, meter stick
	Human Circle	49	6.14		Pieces of string (5-10 feet) for each participant, chalk

Topic: The van Hiele Theory of Geometric Thought

Description: The van Hiele theory of geometric thought describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. In this first session, the participants will explore the van Hiele levels of geometric thought by doing triangle sorts and comparing their sorts to those performed by elementary students. The sorting task is appropriate for all ages and levels of students. It can serve as an activity to help students advance their level of understanding as well as an assessment tool that can inform the teacher at what van Hiele level the student is thinking.

Related SOL: 6.14, 6.15, 7.9, 7.10, 7.11, 7.12

Activity: The van Hiele Levels of Geometric Thought

Format: Large Group Lecture and Small Group Activity

Objectives: Participants will be able to describe the developmental sequence of geometric thinking according to the van Hiele theory of geometric thought and activities suitable for each level. In addition, participants will be able to assess the van Hiele levels of their students.

Related SOL: 6.14, 6.15

Materials: Paper triangles, cut out and placed in a plastic baggy or manila envelope (see Activity Sheet for Triangle Sorting Pieces). You will need at least one set of triangles for every three participants.

Time Required: Approximately 1 hour

Background: To Trainer (for lecture):
After observing their own students, Dutch teachers P.M. van Hiele and Dina van Hiele-Geldof described learning as a discontinuous process with jumps that suggest "levels." They identified five sequential levels of geometric understanding or thought:

1) Visualization
2) Analysis
3) Abstraction
4) Deduction
5) Rigor

Clements and Battista (1992) proposed the existence of a Level 0 that they called Pre-recognition.

In Kindergarten through grade two most students will be at Level 1. By grade three, students should be transitioning to Level 2. If the content in the Virginia Standards of Learning is mastered, students should attain Level 3 by the end of sixth grade. Level 4 is usually attained by students who can prove theorems using deductive techniques. One problem is that most current textbooks provide activities requiring only Level 1 thinking up through sixth grade and teachers must provide different types of tasks to facilitate the development of the higher levels of thought.

Directions:

1) Participants should study the The van Hiele Levels Explanation Sheet.

2) Turn to the Additional Points Explanation Sheet. Note for Point 1 that the levels are hierarchical. Students cannot be expected to write a geometric proof successfully unless they have progressed through each level of thought in turn. At Point 2, college students and even some teachers have been found who are at Level 1, while there are middle schoolers at Level 3 and above. (If the content in the SOL is mastered, students should attain Level 3 by the end of sixth grade.) As an example of an experience that can impede progress (Point 3), think of the illustration of the teacher who knew that the relationship between squares and rectangles was a difficult one for her fourth-graders so she had them memorize, "Every square is a rectangle, but not every rectangle is a square." When tested a few weeks later, half the students remembered that a square is a type of rectangle, while the other half thought that a rectangle was a type of square. It was almost impossible for these students to learn the true relationship between squares and rectangles because every time they heard the words square and rectangle together, they insisted on relying on their memorized sentence rather than on the properties of the two types of figures.

3) Continue on to Properties of Levels. As an example of separation, consider the meaning of the word "square." When a teacher thinking at Level 3 or above says "square," the word conveys the properties and relationships of a square: having four congruent sides; having four congruent angles; having perpendicular diagonals; and being a type of polygon, quadrilateral, parallelogram, and rectangle. To a student thinking at Level 1, the word "square" will only evoke an image of something that looks like a square such as a CD case or first base. The same word is being used, but it has an entirely different meaning to the teacher and the student. The teacher must keep in mind what the meaning of the word or symbol is to the student and how the student thinks about it. For Attainment, it is important to note that there are five phases of learning that lead to understanding at the next higher level.

4) Divide participants into small groups. Distribute the sets of cutout triangles, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them triangles. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they

are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way to sort the flag parts would be to put all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. If needed, you can demonstrate a triangle sort using pieces cut from the Triangle Sorting Pieces Activity Sheet. Have participants sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

5) Ask the participants to describe their sorts. Expect answers like "acute, right, and obtuse triangles" or "scalene, isosceles, and equilateral." Have them compare their sorts with those of other groups.

6) Ask them how they think their students would sort these figures. Refer to Sample Student Sort Sheets and ask the participants to conjecture the criteria used for sorting and the van Hiele level of the sorter. Sample Student Sort 1 is a low Level 1 sort where the student is sorting strictly by size and may not even know that the figures are triangles. Sample Student Sort 2 is another Level 1 sort. Here the student thinks that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Sample Student Sort 3 is another Level 1 sort. This student also believes that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Additionally, this student recognized the figures with right angles or "corners" as a separate category. The Sample Student Sort 4 is at least a Level 2 or 3 sort in which the sorter focuses on the lengths of the sides, a criterion that separates the figures into categories that overlap. The student has actually sorted into groups with no sides the same length, two sides the same length, and all sides the same length. It is unclear whether the student knows that equilateral triangles are a type of isosceles triangle. The Sample Student Sort 5 focuses on parts of the figures and so is a Level 2 sort, but the student does not have the vocabulary to adequately describe the figures. The Sample Student Sort 6 is similar to the Sample Student Sort 4, but the word "Perfect" is incorrect and indicates that the student may be thinking more of the figure as a whole rather than of the individual parts. This sort is probably Level 2
Note: Descriptions written on the Sample Student Sort Sheets are the words of the student who performed the sort.

Explanation Sheet: The van Hiele Levels

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of the figure. A student should recognize and name figures, and distinguish a given figure from others that look somewhat the same. “I know it's a rectangle because it looks like a door and I know that the door is a rectangle."

Level 2: Analysis. Properties are perceived, but are isolated and unrelated. A student should recognize and name properties of geometric figures. “I know it's a rectangle because it is closed, it has four sides and four right angles, opposite sides are parallel, opposite sides are congruent, diagonals bisect each other, adjacent sides are perpendicular,...”

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. “I know it’s a rectangle because it's a parallelogram with right angles."

Level 4: Deduction. The student can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. A student should be able to supply reasons for steps in a proof.

Level 5: Rigor. The standards of rigor and abstraction represented by modern geometries characterize Level 5. Symbols without referents can be manipulated according to the laws of formal logic. A student should understand the role and necessity of indirect proof and proof by contrapositive.

Explanation Sheet: Additional Points

1. The learner cannot achieve one level without passing through the previous levels.

2. Progress from one level to another is more dependent on educational experience than on age or maturation.

3. Certain types of experiences can facilitate or impede progress within a level or to a higher level.

Properties of Levels

Adjacency: What was intrinsic in the preceding level is extrinsic in the current level.

Distinction: Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Separation: Two individuals reasoning at different levels cannot understand one another.

Attainment: The learning process leading to complete understanding at the next higher level has five phases: inquiry/information, directed orientation, explication, free orientation, and integration.

Phases of Learning

Inquiry/Information: Gets acquainted with the working domain (e.g., examines examples and non-examples).

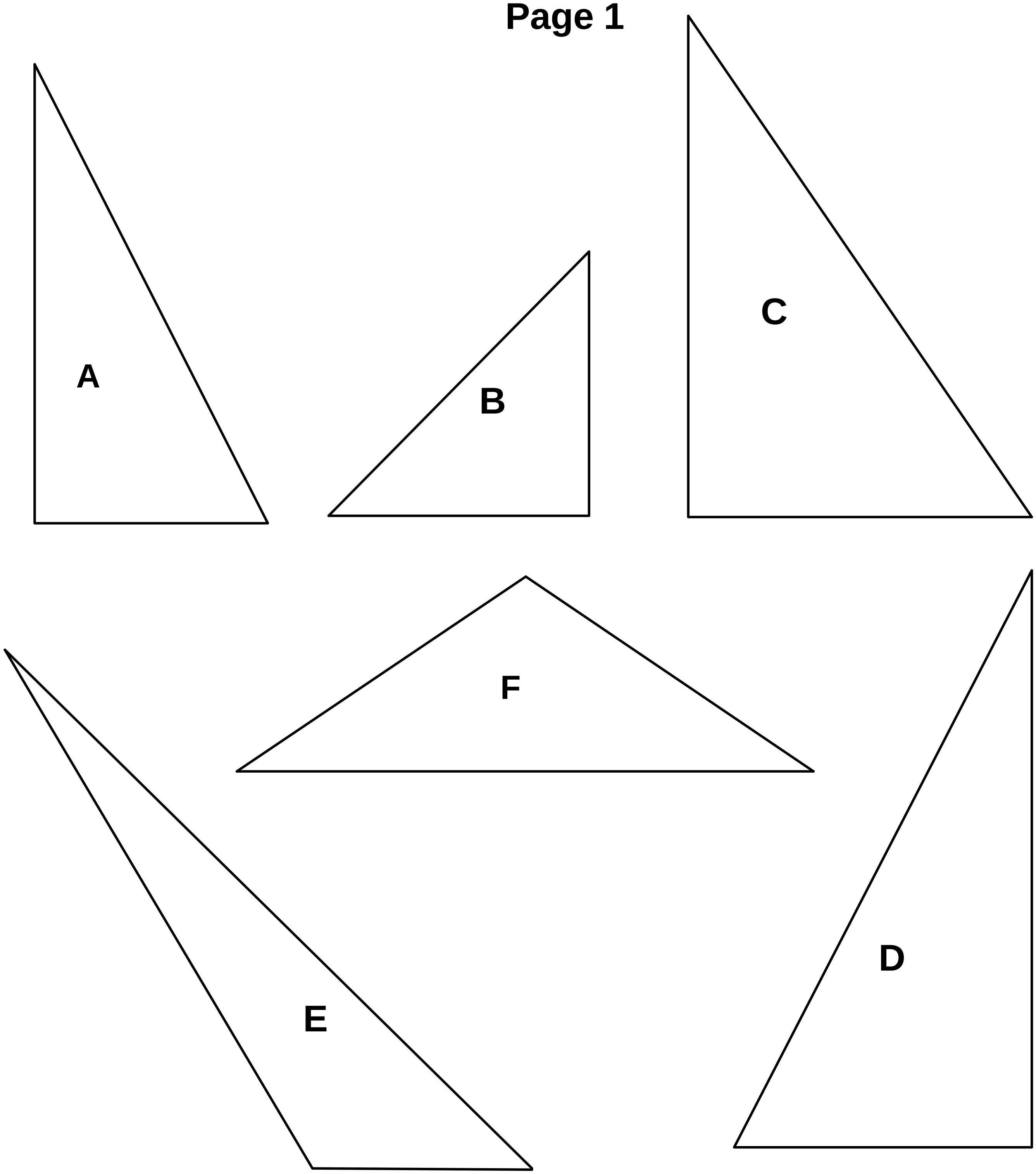
Guided orientation: Does tasks involving different relations of the network that is to be formed (e.g., folding, measuring, looking for symmetry).

Explication: Becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures).

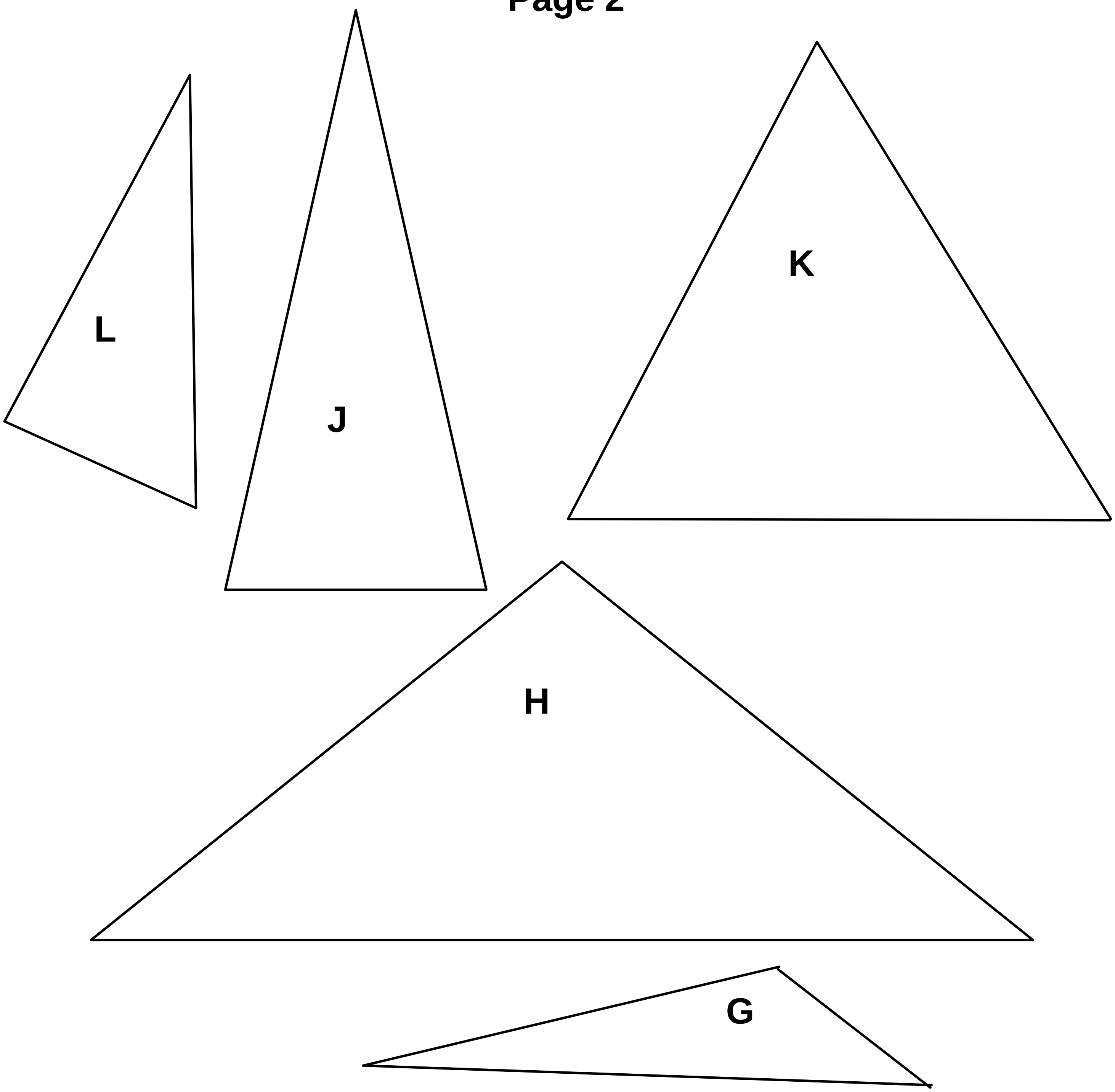
Free orientation: Learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g., knowing properties of one kind of figure, investigating these properties for a new figure, such as kites).

Integration: Summarizes all that has been learned about the subject, then reflects on actions, and obtains an overview of the newly formed network of relations now available (e.g., properties of a figure are summarized).

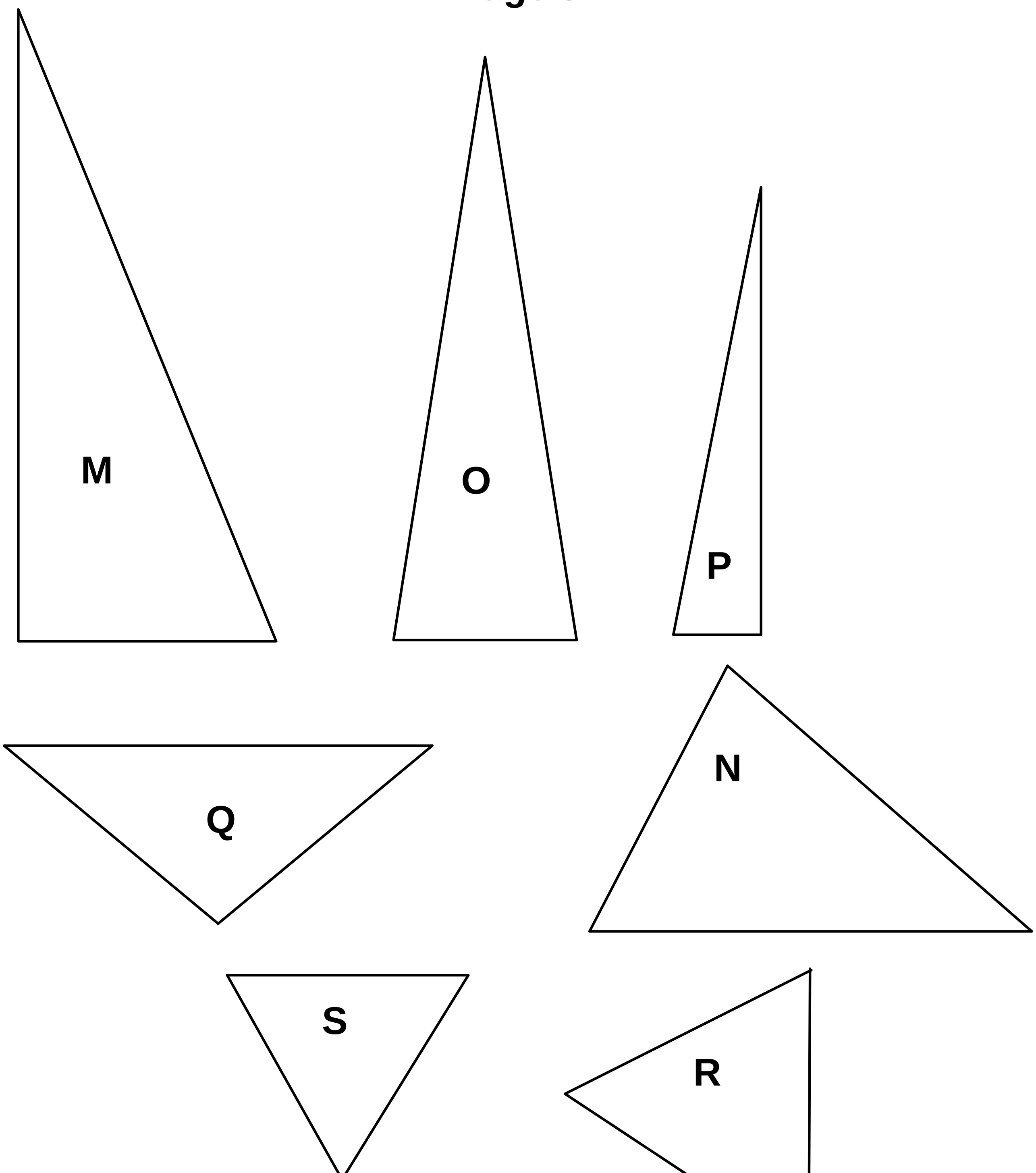
Triangle Sorting Pieces
Page 1



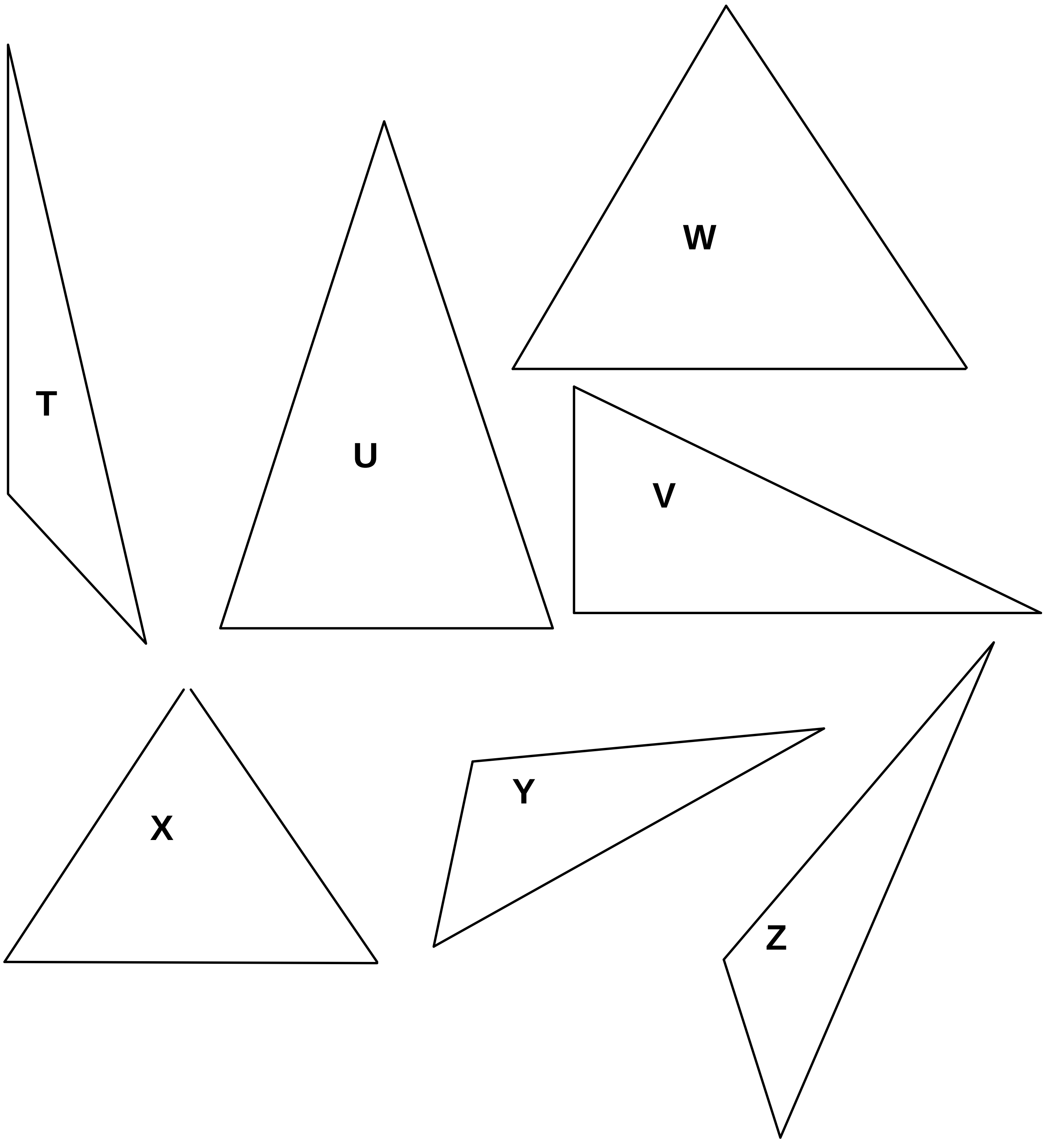
Triangle Sorting Pieces
Page 2



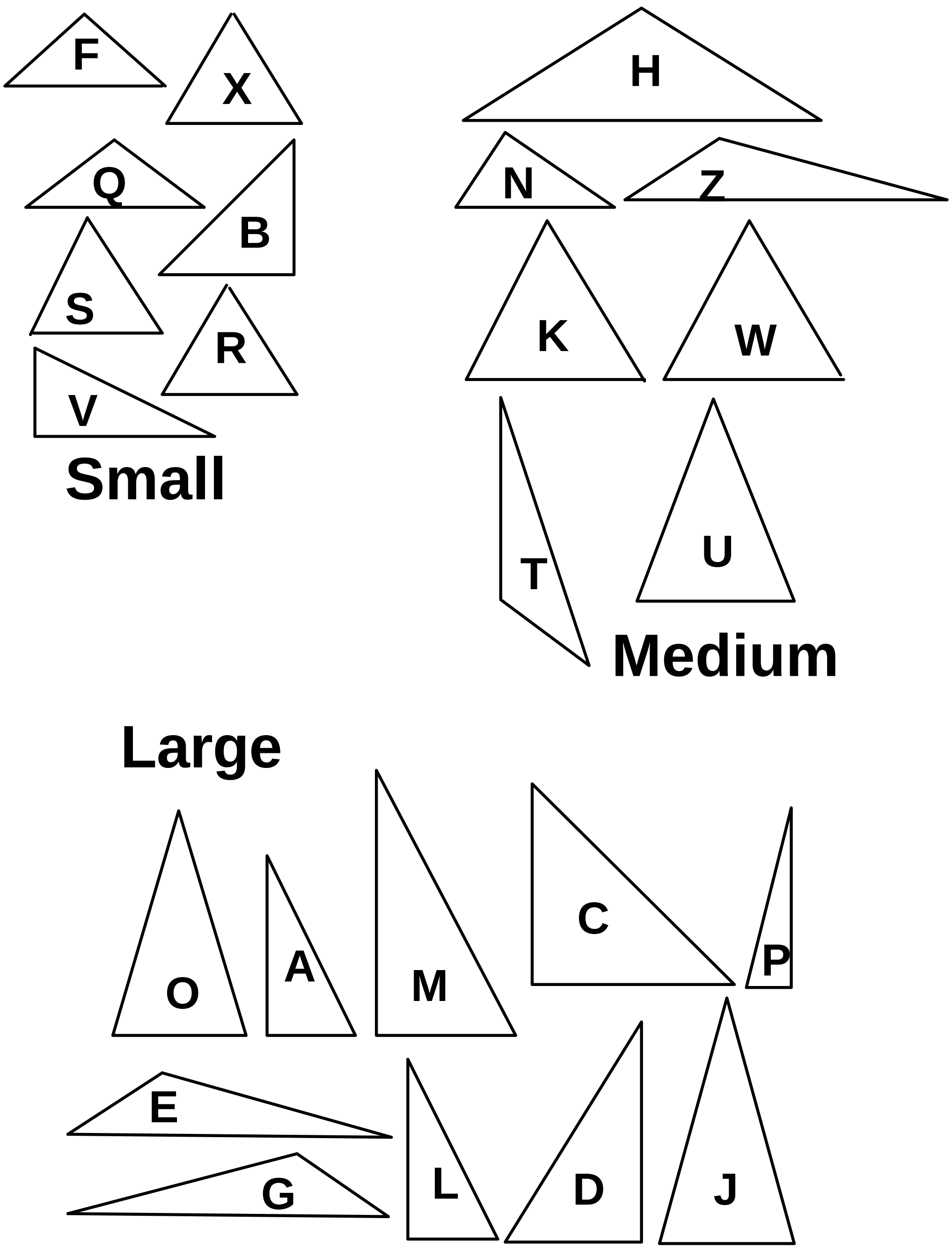
Triangle Sorting Pieces
Page 3



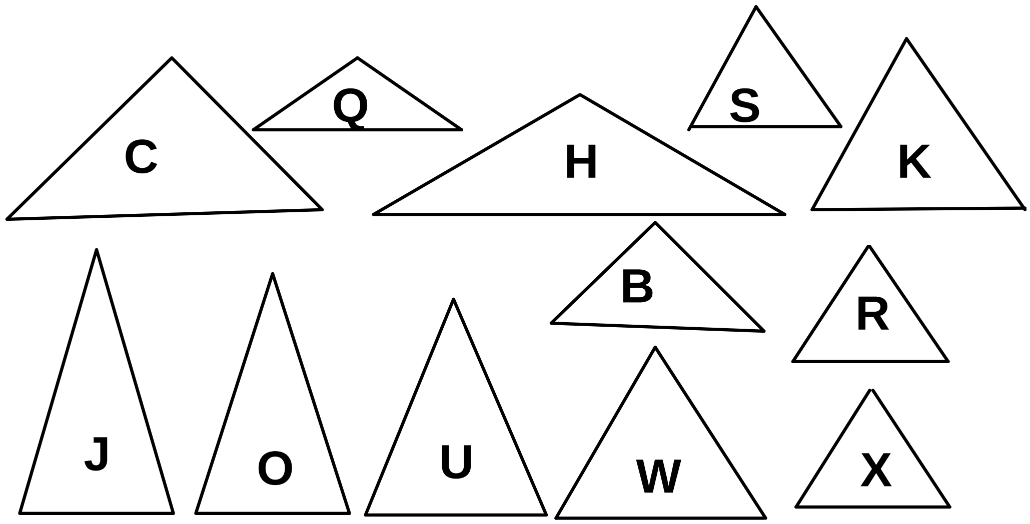
Triangle Sorting Pieces
Page 4



Sample Student Sort 1

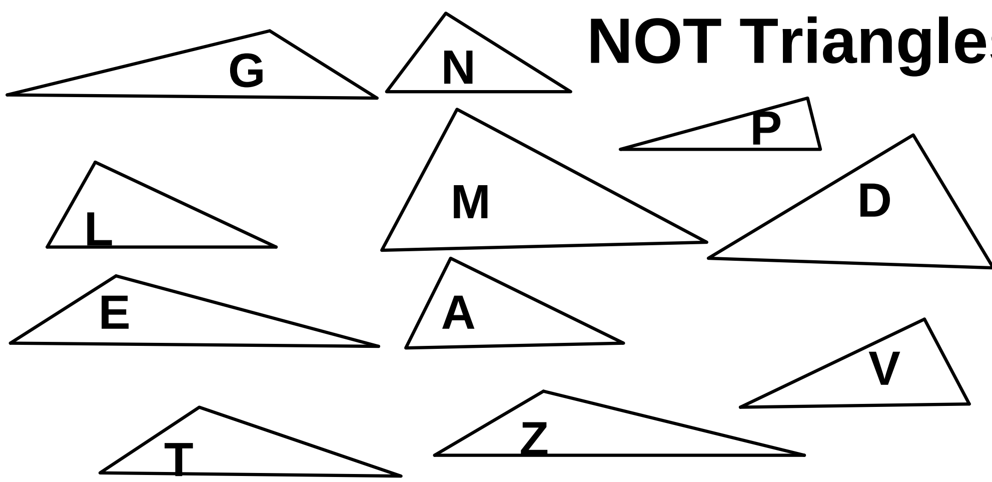


Sample Student Sort 2

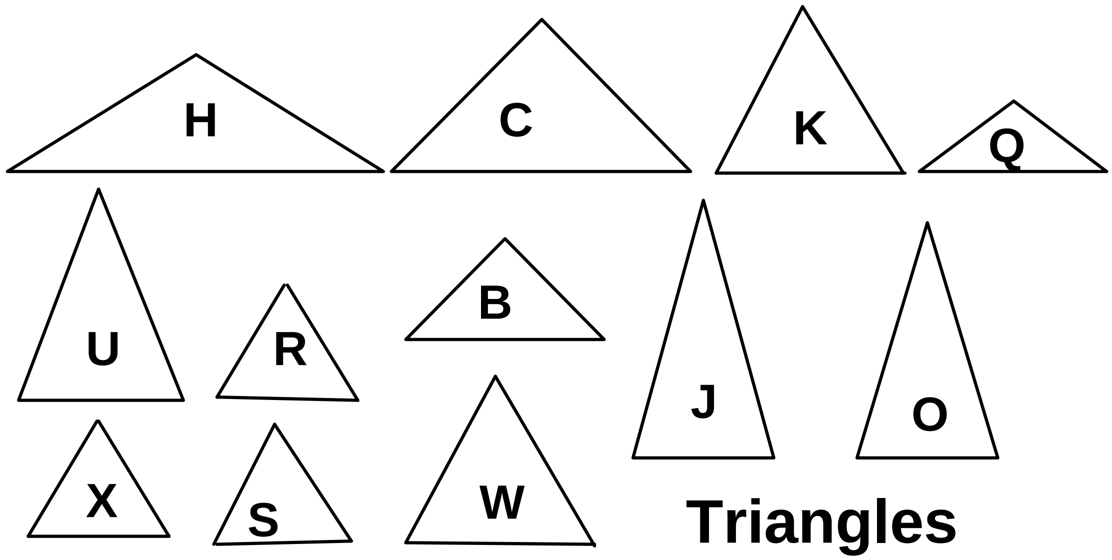


Triangles

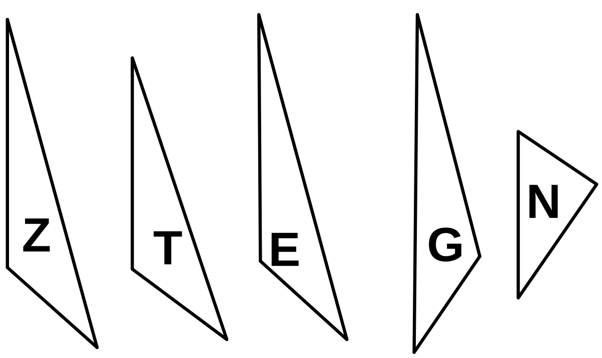
NOT Triangles



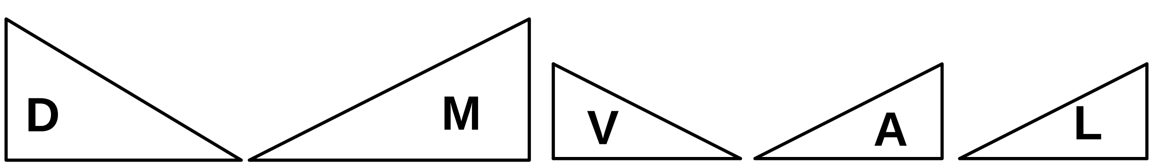
Sample Student Sort 3



Triangles

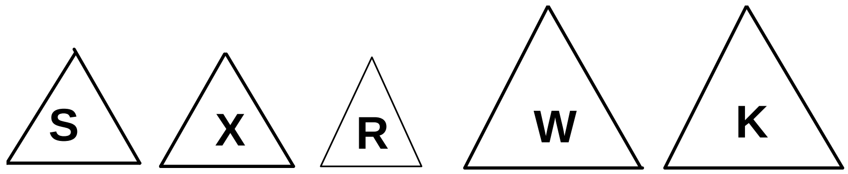


Look alike...
N is smaller...
NOT triangles

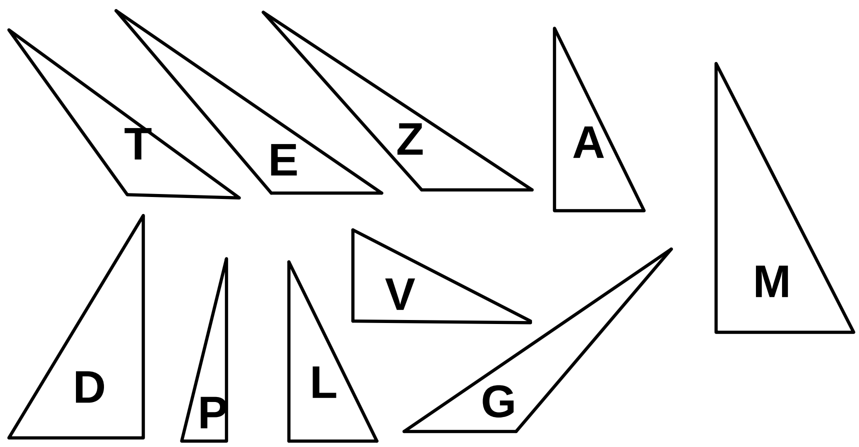


Look like ramps... NOT triangles

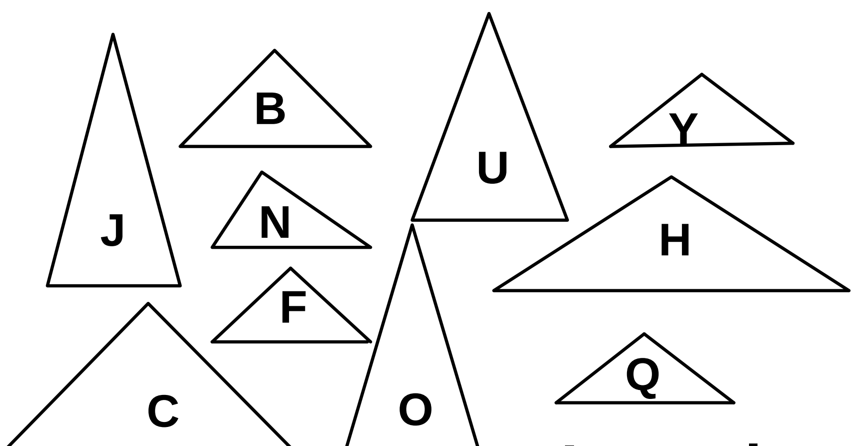
Sample Student Sort 4



Equilateral

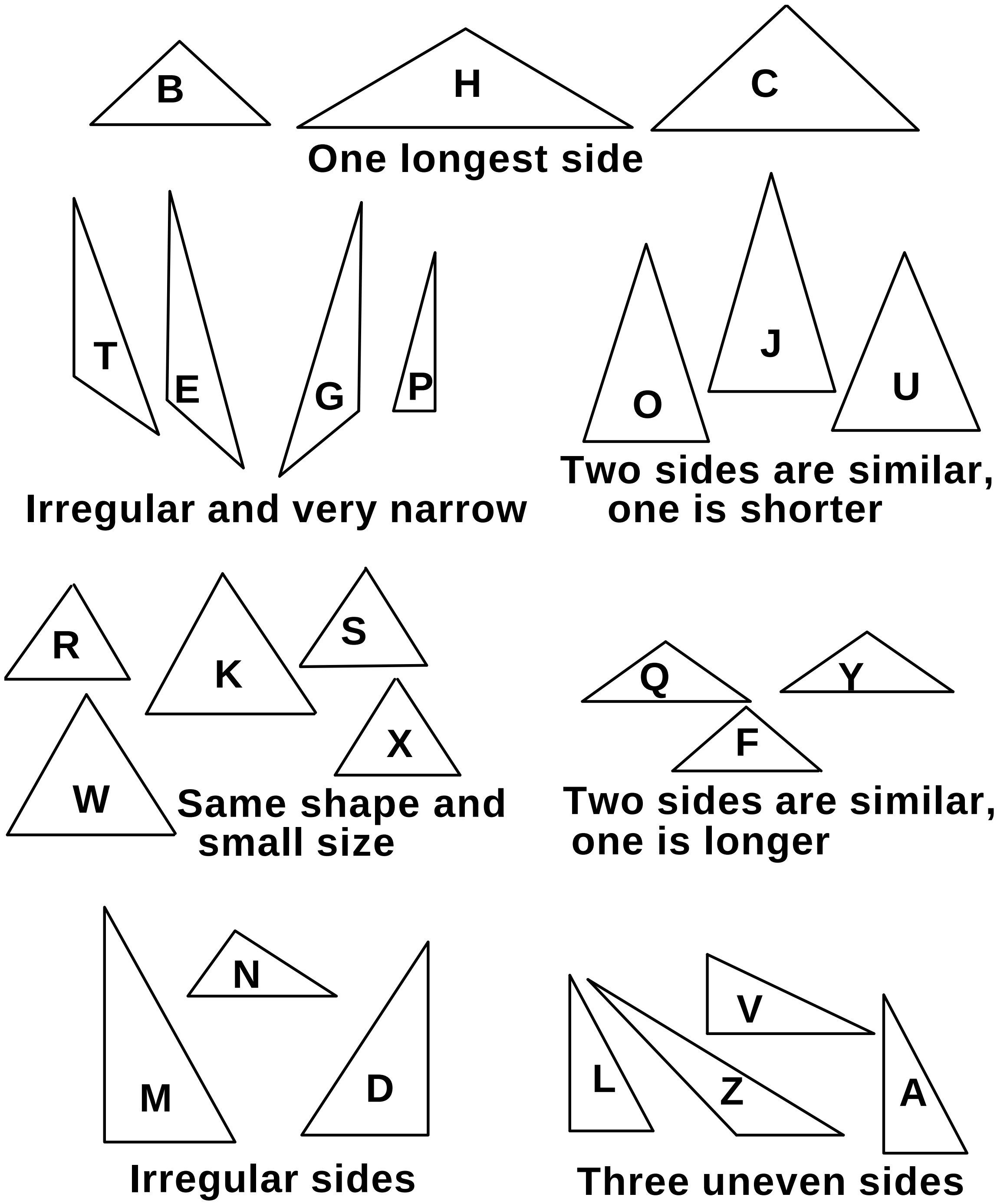


Scalene

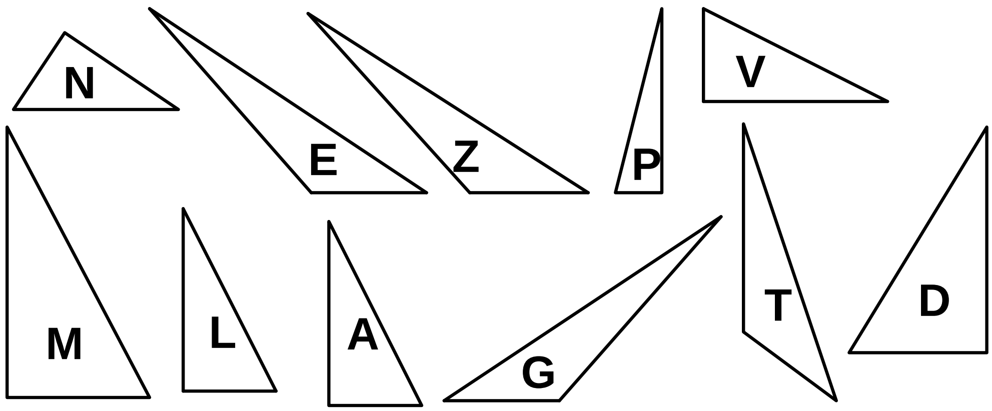


Isosceles

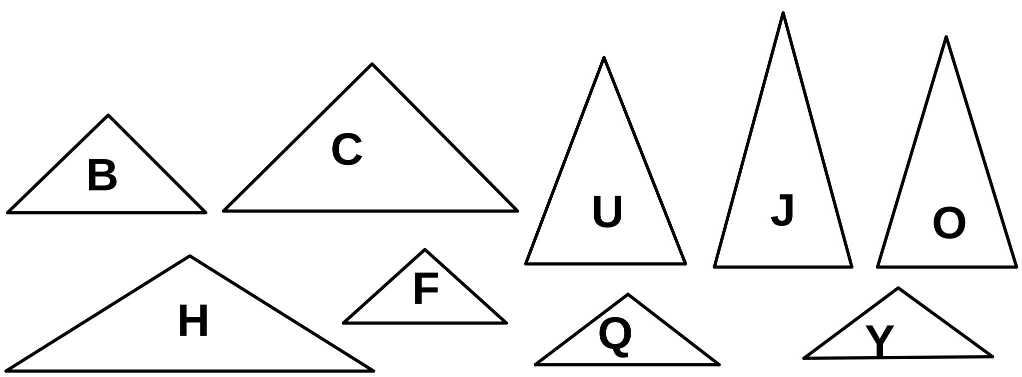
Sample Student Sort 5



Sample Student Sort 6

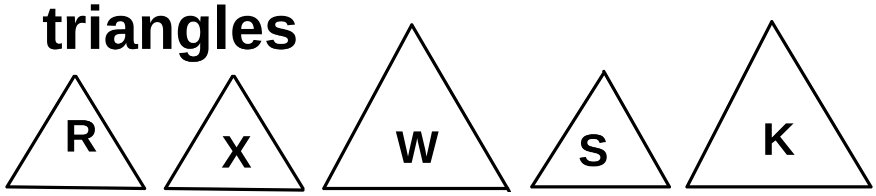


Every side has a different size



Two sides equal, third side smaller or larger

"Perfect" triangles



Topic: Quadrilaterals and their Properties

Description: Participants will explore quadrilaterals and their properties through the use of various manipulatives such as sorting pieces and geo-strips. The sequence of activities is designed to facilitate an increase in a learner's van Hiele level of thinking about quadrilaterals from Level 1 to Level 3. First, the participants learn how to determine the van Hiele levels of their own students by analyzing how they sort a set of quadrilateral pieces. Then they play the game "What's My Rule?" to develop the ability to classify quadrilaterals by various attributes and to focus on more than one attribute at a time. The participants also construct parallelograms, rectangles, rhombi, and squares using D-stix, geo-strips, toothpicks, or other manipulatives and make observations while the figures are flexed (Level 2). Finally, the participants identify relationships between parallelograms, rectangles, rhombi, squares, trapezoids, kites, and darts through a lab that culminates in the creation of a quadrilateral family tree (Level 3).

Although these activities are presented with quadrilaterals, most of them are easily adapted to triangles and other polygons.

Related SOL: 6.14, 7.9

Activity: Quadrilateral Sort

Format: Small Group/Large Group

Objectives: After performing their own sorts, participants will be able to distinguish the way students at various van Hiele levels of geometric thought may sort quadrilaterals.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. You will need at least one set of quadrilaterals for every three participants.

Time Required: Approximately 20 minutes

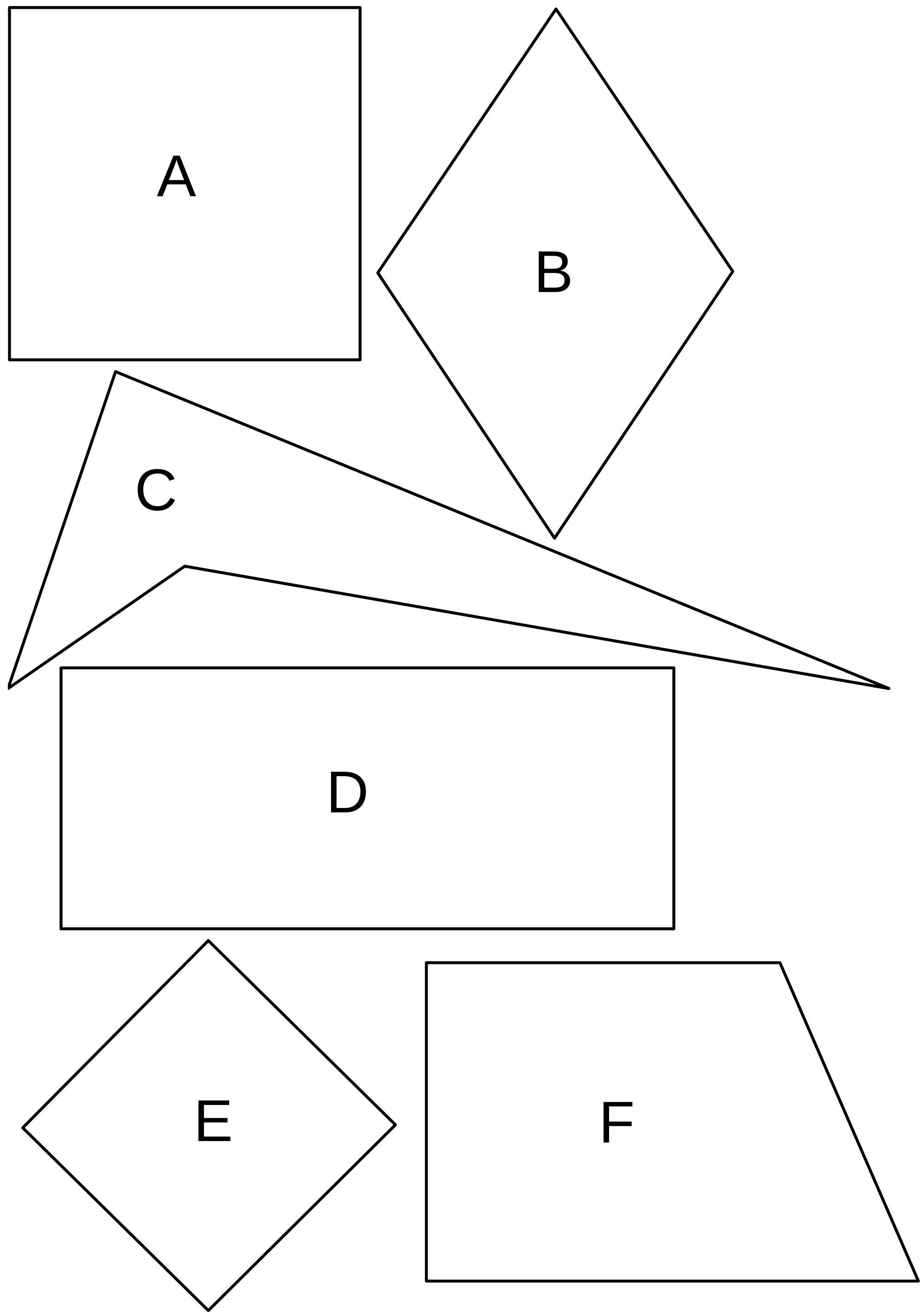
Directions:

1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them quadrilaterals. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way the flag parts could be grouped would be all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. Have them sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

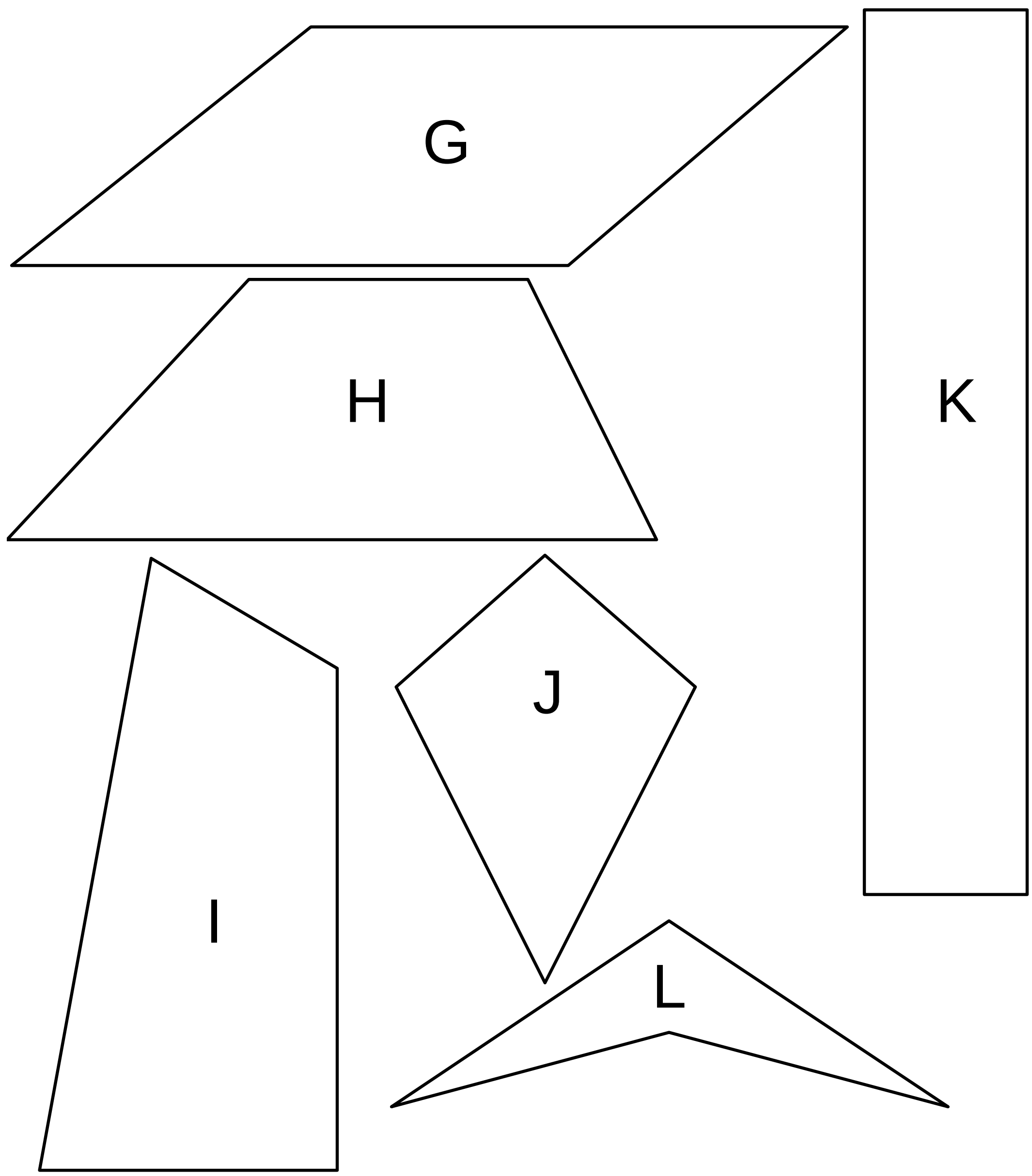
2) Ask the participants to describe their sorts. Have them compare their sorts with those of other groups.

3) Ask them how they think their students would sort these figures.

Quadrilateral Sorting Pieces



Quadrilateral Sorting Pieces
page 2



Activity: What's My Rule?

Format: Small Group/Large Group

Objectives: After playing the game, participants will classify quadrilaterals by various attributes. In children, this game develops the ability to attend to more than one characteristic of a figure at the same time.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). You will need at least one set of quadrilaterals for every three or four participants. "What's My Rule?" Activity Sheet

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, one set per group.

2) Display "What's My Rule?" Activity Sheet and review the rules of the game. One participant in each group is the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.

3) At any time, the players can call "stop" and guess the rule. After the correct rule identification, the player who figured out the rule becomes the sorter. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty. The winner is the first one to accumulate ten points.

WHAT'S MY RULE?
Rules

1. Choose one player to be the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.

2. At any time, the players can call "stop" and guess the rule. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty.

3. After the correct rule identification, the player who figured out the rule becomes the sorter.

4. The winner is the first one to accumulate ten points.

Activity: Quadrilateral Properties Laboratory

Format: Small Group/Large Group

Objectives: Participants will construct parallelograms, rectangles, rhombi, and squares, using D-stix, geo-strips, or toothpicks and marshmallows. Participants will identify the properties of the constructed figures.

Related SOL: 6.14, 7.9

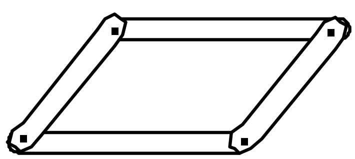
Materials: One of the following per participant: D-stix, geo-strips, or toothpicks cut into two different lengths and marshmallows; square corner (the corner of an index card or book); Types of Quadrilaterals Activity Sheet

Time Required: Approximately 20 minutes

Directions:

1) Divide the participants into small groups and direct each group to experiment as you ask questions. Be sure to model constructing the quadrilaterals and flexing them.

2) Have the participants pick two pairs of congruent segments and connect them as shown below. Have them flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of the measures of the angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What do you notice about the opposite sides of this quadrilateral? (They remain parallel and congruent.)

A **parallelogram** is a quadrilateral with both pairs of sides parallel.

- What is the sum of the measures of the interior angles of this quadrilateral? (360°)

- What do you notice about the opposite angles? (Congruent)

Note to Trainer: Some participant will likely turn the strips so that they cross, forming two triangles. If no one does, you should. Ask if this figure is a polygon. Elicit from the group what the essential elements of a polygon are, i.e.:

a) composed of line segments;
b) simple (the segments don't cross);
c) closed; and
d) lies in a plane (e.g., if you take a wire square and twist it so that it isn't flat, it is no longer a polygon).

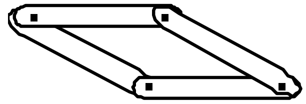
3) Make one of the angles a right angle. You can use the square corner to check your accuracy.
   Ask:
   - What happens to the other angles? (They become right angles.)
   - Will this always be true when you make one angle of a parallelogram a right angle? (Yes)
   - How do you know? (The sum of the measure of the angles in a parallelogram is 360°. One angle measures 90°. Its opposite angle must measure the same or 90°. Subtracting these two angles from 360°, the remaining two angles, which are congruent since they are opposite angles in a parallelogram, must have a total measure of 180°. Therefore, each angle measure is 90°. Note: This is Level 3 thinking.)
   - Is it still a parallelogram? (Yes)
   - Is it still a quadrilateral? (Yes)
   - Is it still a polygon? (Yes)
   - What other name, besides polygon, quadrilateral, and parallelogram, can be given to it now? (Rectangle)

   A **rectangle** is a parallelogram with four right angles.

4) Make a parallelogram that has all four sides equal in length. What is another name for this parallelogram? (Rhombus)

   A **rhombus** is a parallelogram with four congruent sides.

5) Flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What is the sum of the measures of the interior angles of this quadrilateral? (360°)
- What do you notice about the opposite angles? (Congruent)
- Is it still a quadrilateral? (Yes)
- Is it still a polygon? (Yes)

6) Make one of the angles of this rhombus a right angle, checking with your square corner.
   Ask:
   - What happens to the other angles? (All right angles)
   - Is it still a parallelogram? (Yes)
   - What other name, besides polygon, quadrilateral, parallelogram, and rhombus, can be given to this new figure? (Square)

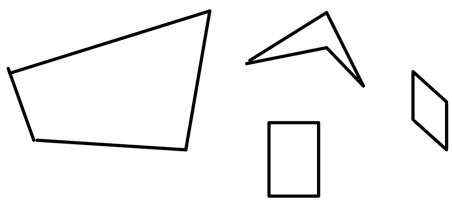
   A **square** is a parallelogram with four congruent sides and four right angles.

   Ask:
   - Is it a rectangle? (Yes)
   - How do you know? (It has four right angles.)

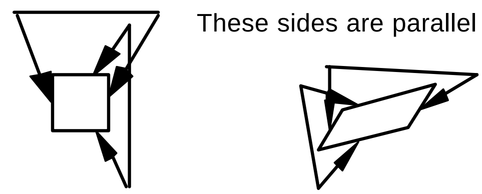
7) Distribute Types of Quadrilaterals Activity Sheet and discuss the definitions for quadrilateral, parallelogram, rectangle, rhombus, and square. Discuss the examples of each, noticing their orientations and how each example fits the definition even though they are not necessarily the stereotypical figure usually seen. Discuss the implications for teaching a Level 1 student who recognizes figures by comparing them to a known figure. This type of student might describe a rectangle by saying, "I know it's a rectangle because it looks like a door."

TYPES OF QUADRILATERALS

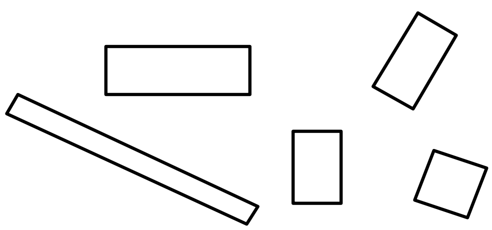
A **quadrilateral** is a **four**-sided polygon.



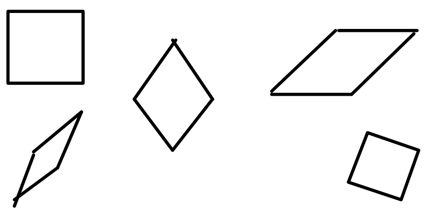
A **parallelogram** is a quadrilateral with both pairs of **opposite** sides **parallel**.



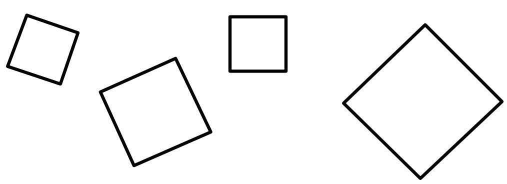
A **rectangle** is a quadrilateral with **four right angles**.



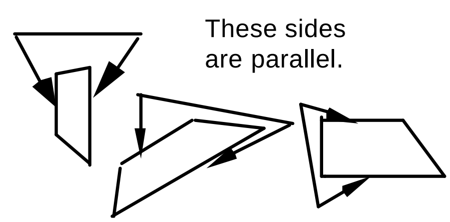
A **rhombus** is a quadrilateral with **four sides congruent**.



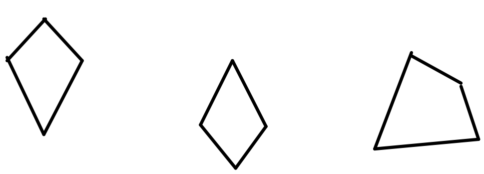
A **square** is a quadrilateral with **four right angles** and **four congruent sides.**



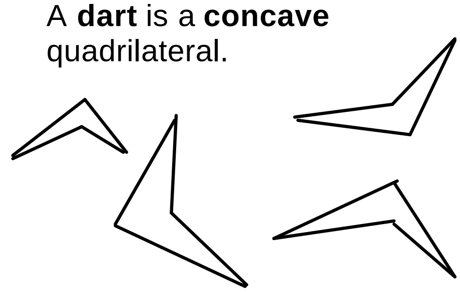
A **trapezoid** is a quadrilateral with **exactly one** pair of parallel sides.



A **kite** is a convex quadrilateral with two distinct pairs of **adjacent congruent sides**.



A **dart** is a **concave** quadrilateral.



Activity: Quadrilateral Sorting Laboratory

Format: Small Group/Large Group

Objectives: Participants will record which quadrilaterals meet the various descriptions listed in the properties table, determine which sets are identical and are subsets of one another, attach labels to each category, and create a quadrilateral family tree.

Related SOL: 6.14, 7.9

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet, Quadrilateral Family Tree Activity Sheet

Time Required: Approximately 30 minutes

Directions:

1) Distribute Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet and the Quadrilateral Family Tree Activity Sheet. Divide the participants into small groups and direct each group to experiment and answer the questions, using their quadrilateral sorting pieces.

2) After the participants have filled out the Quadrilateral Table Activity Sheet, have pairs of groups compare their answers, and reconcile any discrepancies.

3) Have the participants continue with Steps 5-10. Refer to the Quadrilateral Table as needed while discussing the results of #10.

4) For Step 11 the participants can construct the family tree as small groups or as a large group. Discuss various possibilities for the entries.

Quadrilateral Sorting Laboratory

Directions:

1) Spread out your quadrilateral pieces with the letters facing up so you can see them.

2) Find all of the quadrilaterals having four right angles. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.

3) Consider all of the quadrilaterals again. Find all of the quadrilaterals having exactly one pair of parallel sides. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.

4) Continue in this manner until the Quadrilateral Table is complete.

5) Which category is the largest? What name can be used to describe this category?

6) Which lists are the same? What name can be used to describe quadrilaterals with these properties?

7) Are there any lists that are proper subsets of another list? If so, which ones?

8) Are there any lists that are not subsets of one another that have some but not all members in common? If so, which ones?

9) Which lists have no members in common?

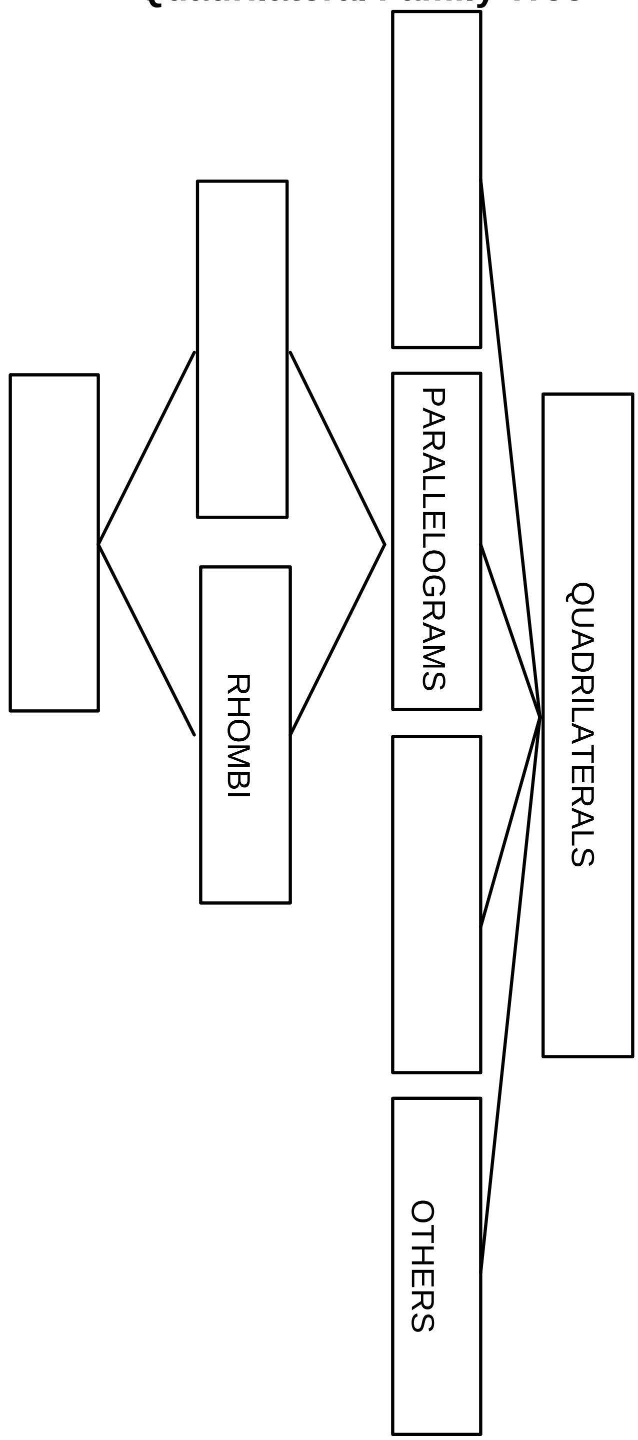
10) Label each of the categories in the Quadrilateral Table with the most specific name possible using the labels kite, quadrilateral, parallelogram, rectangle, rhombus, square, and trapezoid. For example, #1 - a quadrilateral that has four right angles is a rectangle. (Having four right angles is not enough to make it a square; it would need four congruent sides as well.)

11) Compare your results to that of the other groups. Then fill out the family tree by inserting the names kites, rectangles, squares, and trapezoids into the appropriate places on the diagram.

Quadrilateral Table

1. has four right angles
2. has exactly one pair of parallel sides
3. has two pairs of opposite sides congruent
4. has four congruent sides
5. has two pairs of opposite sides parallel
6. has no sides congruent
7. has two pairs of adjacent sides congruent, but not all sides congruent
8. has perpendicular diagonals
9. has opposite angles congruent
10. is concave
11. is convex
12. its diagonals bisect one another
13. has four sides
14. has four congruent angles
15. has four congruent sides and four congruent angles

Quadrilateral Family Tree



Topic: Plane Figures and their Properties

Description: Participants will explore plane figures and their properties through the use of various manipulatives such as geoboards, and geo-strips. A writing exercise is also included. Similar, congruent, and non-congruent plane figures are constructed and examined.

Related SOL: 6.11, 6.13, 6.14, 6.15, 7.7, 7.9, 7.10

Activity: Polygons and the Geoboard

Format: Large Group

Objectives: The participant will correctly manipulate the rubber band on the geoboard to make polygons with an increasing number of sides.

Related SOL: 6.14, 6.15, 7.9, 7.10

Materials: 11 x 11 geoboards, rubber bands

Time Required: Approximately 15 minutes

Directions:

1) Remind participants of appropriate use of the geoboards and ensure that everyone has a board and a rubber band. Ask participants to make a triangle on their geoboard.

2) Ask the participants to modify the shape on their geoboard by adding one more side. Continue from a triangle to a quadrilateral to a pentagon, and so forth.

3) Have the participants compare their shapes on the geoboards with each other. Use this opportunity to talk about the fact that all four-sided figures are quadrilaterals — some are squares, some trapezoids, and so forth. This is a good time to reinforce the idea of common properties (all quadrilaterals have four sides) and distinguishing properties (all squares have four congruent sides and four right angles.)

4) Challenge participants to make a polygon with the largest number of sides. Is the answer different if the polygon is convex or concave?

Activity: A Country Mile

Format: Individual or Small Group

Objectives: Participants will find and support a reasonable response to the writing prompt using their knowledge of the perimeter of polygons and circles.

Related SOL: 6.11, 6.14, 7.7, 7.9, 7.10

Materials: A Country Mile Writing Prompt Activity Sheet, geoboards and rubber bands (optional)

Time Required: Approximately 30 minutes plus writing time

Directions:

1) Discuss the nature of an open response item with participants. Let them know that these questions may have more than one correct answer. When used with students, work should be graded for coherence of thought, careful presentation, reasonableness of the answer, and support of the answer given.

2) The prompt as presented has very few hints included. The questions below may help you guide participants who have difficulty getting started.
   - How long is a day from sunup to sundown? Is it the same all year?
   - What shape includes the most area for a given perimeter? Your geoboard may help you explore this question.
   - How far can you walk in a day? Don't forget about rest breaks, lunch break, etc.

3) Discuss the evaluation criteria with students before they write up their solution. This is a good assignment for collaboration with a language arts teacher for the revising and editing of the writing.

Reference: Danielson, C. (1997). *A Collection of Performance Tasks and Rubrics: Middle School Mathematics*. Larchmont, NY: Eye on Education.

A Country Mile

In an old folktale, a poor peasant is offered as much land as he can walk around from sunup to sundown. If you were given that offer, how much land could you claim? What shape would it be?

Your answer should include these items:

- Drawings of different possible shapes with area and perimeters;

- A clear presentation of your calculation methods
  - the length of the day
  - the distance you can walk
  - the area you claim;

- Clearly labeled work; and

- Neat and accurate writing.

Activity: Similarity and Congruence with Geostrips

Format: Individual or Pairs

Objectives: Participants will create similar and congruent figures using geostrips and compare the figures to determine which properties are the same and which are different.

Related SOL: 6.14, 6.15, 7.9, 7.11

Materials: Geostrips for each student or pair of students, protractors or angle rulers

Time Required: Approximately 30 minutes

Directions:

1) Ask the participants to define congruent figures. Guide the discussion towards a definition that includes the fact that all properties of the two figures are exactly the same and that the two figures would match perfectly if laid one on top of the other. Be sure to include proper vocabulary (side, vertex, angle) in this discussion so that the participants are familiar with the terminology as they construct shapes.

2) Have participants take any three geostrips and make a triangle. Ask them to compare their triangle with the triangle of a partner who used different strips. What do they notice? Look for answers such as the fact that one side or one angle matches (is congruent) but that not all the parts match. These triangles, then, are not congruent figures.

3) Next ask each participant to construct a triangle that is congruent to his/her own triangle. Again, compare the two and notice that each part matches exactly. These triangles are congruent figures.

4) Ask participants to construct a triangle using the shortest red, yellow, and blue strips. Participants should then construct a second triangle using the medium length red strip and the longer blue and yellow strips.

5) Ask participants to compare the two triangles, noticing what is similar and what is different. They should observe that the angles are congruent but the sides are not. Encourage the participants to use other strips from their supply to determine the 2:1 ratio between the two sets of strips.

6) Share with the participants that polygons whose angles are congruent and whose sides are proportional are similar figures. Emphasize that this is a different use of the term similar than we use in everyday English. Students often think similar means “almost alike” even in mathematics. It is important that they realize the term has a precise definition in geometry.

7) This activity can be repeated with other polygons.

Activity: Perimeters and Areas of Similar Triangles

Format: Small Group

Objectives: Participants will describe the relationship between area and perimeter of similar triangles in terms of the relationship of the triangle sides.

Related SOL: 6.11, 7.7, 7.11

Materials: A dynamic geometry software program, Data Chart: Area and Perimeter of Similar Triangles.

Time Required: Approximately 45 minutes

Directions:

1) This exercise assumes participants are familiar with a dynamic geometry software program enough to construct a triangle and measure its area and perimeter.

2) Pose the following question to participants: If two triangles are similar, what is the relationship between their areas? Between their perimeters?

3) Participants can begin thinking about this question by looking at the grid that appears on the Data Chart. Ask participants to name triangles using the marked vertices and to determine if the triangles are similar. When the participants have found two similar triangles, ask them to calculate the area and the perimeter of each triangle.

4) Using a dynamic geometry software program, have participants test their hypothesis on triangles they create. They should use the dilation command within the software to create similar triangles and keep a record of their findings in the chart found in the Data Chart. Emphasize with participants the importance of calculating each ratio in the same way. The side ratio should be Side 1: Side 2, and the area and perimeter ratios should be calculated in the same fashion.

5) If participants do not have access to sketching software, they can create triangles on a geoboard or work from triangles you provide on a handout.

6) As participants discuss their findings, remind them that their examples are supporting evidence, not proof that their hypothesis always holds.

When participants teach this lesson, it is appropriate to collaborate with a science teacher and talk about the difference between mathematical proof (where we show that X always holds true) and scientific proof (where the preponderance of the evidence supports the hypothesis and we can generally make accurate predictions) as this will be an important distinction in more advanced mathematics classes.

Data Chart:
Area and Perimeter
of Similar Triangles

Trial	Side Ratio	Area 1	Area 2	Area Ratio	Perimeter 1	Perimeter 2	Perimeter Ratio

Activity: Making and Using a Hypsometer

Format: Individual or Small Group

Objectives: Participants use the hypsometer and their knowledge of the proportional relationship between similar triangles to determine the height of an object not readily measured directly.

Related SOL: 6.15, 7.11

Materials: For each hypsometer you need a straw, decimal graph paper, cardboard, thread, a small weight, tape, a hole punch, scissors, and a meter stick.

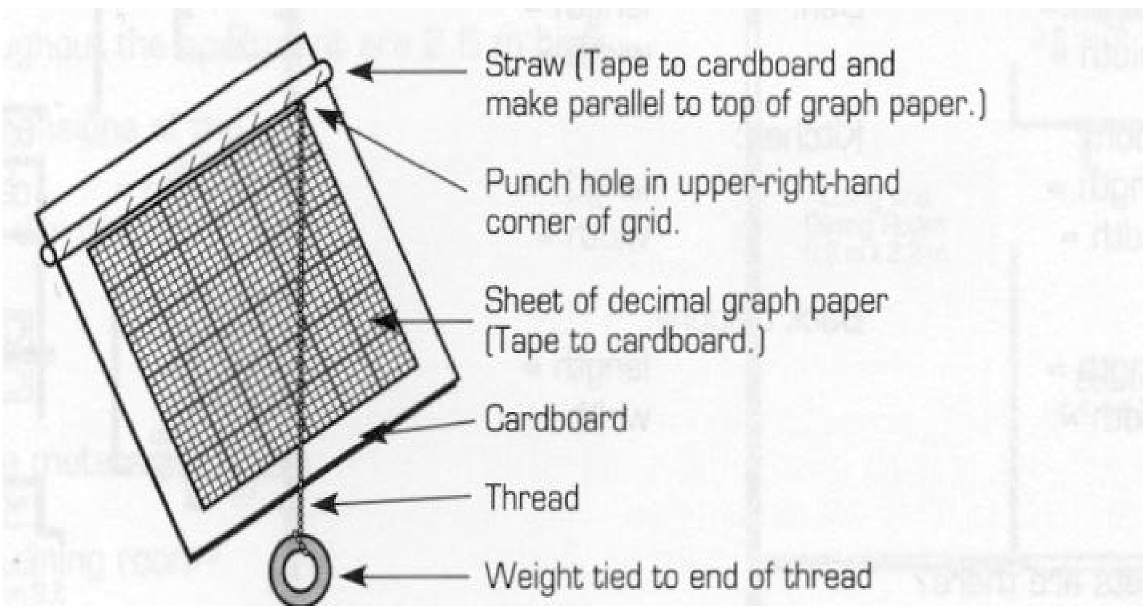
Time Required: Approximately 90 minutes

Directions: To make the hypsometer:

1) Tape a sheet of decimal graph paper to a piece of cardboard.

2) Tape the straw to the cardboard so that it is parallel to the top of the graph paper.

3) Punch a hole in the upper right corner of the grid. Pass one end of the thread through the hole and tape it to the back of the cardboard. Tie the weight to the other end of the thread.

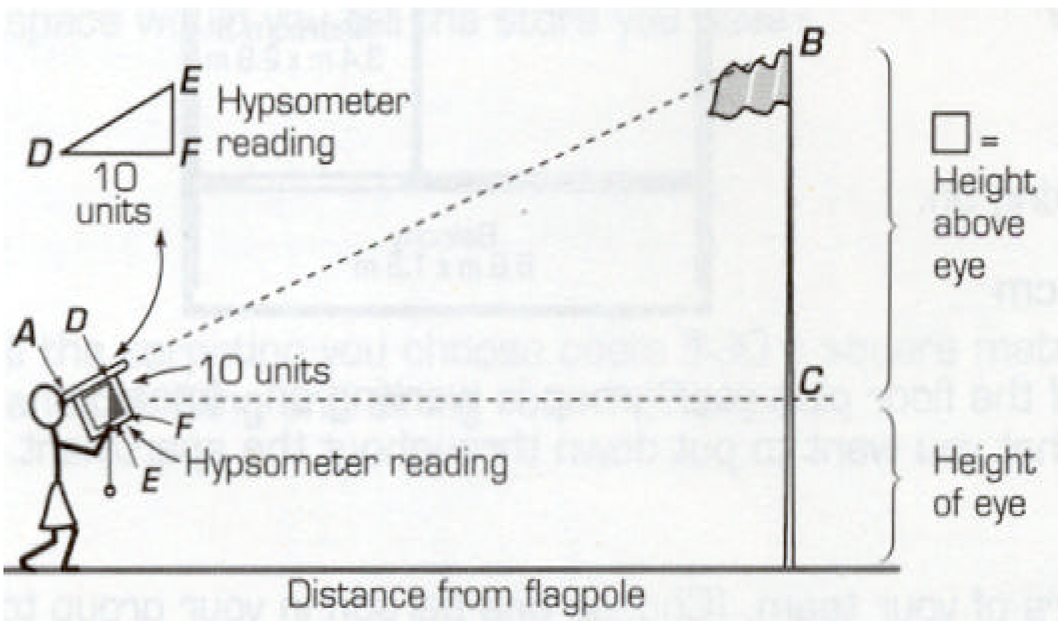


<u>To use the hypsometer</u>:

4) Use a meter stick to measure the height of your eye and the distance to the object you wish to measure.

5) Look through the straw at the top of the object you wish to measure. Use your finger to hold the string where it hangs so you can record the hypsometer reading. Be careful not to move your finger when you take the hypsometer from your eye to read the measurement!

6) To find the height of the flagpole, recognize that triangles ABC and DEF are similar. Thus, BC can be found using the following ratio — AC:DF::BC:EF.



<u>Reference</u>: NCTM Addenda Series, *Measurement in the Middle Grades*

Activity: Human Circle

Format: Large Group

Objective: Participants will develop a definition of a circle and a sphere after creating a human circle.

Related SOL: 6.12

Materials: Enough pieces of string of equal length, for each participant but one; chalk (optional)

Time required: Approximately 15 minutes

Directions:

1) Clear a space larger than twice the length of the string cut or go outside to the playground or parking lot.

2) Choose one participant to be the center of the circle.

3) Have this center person hold the ends of all the strings in one hand, making a fist with all the strings coming out of the top. This person should crouch down, with their arm held over their head.

4) Have every other person take an end of string and back up so the string is taut, spacing themselves around the center person in all directions.

5) (Optional) Draw a circle with chalk on the ground or floor to approximate the circle created by the humans.

6) Discuss the circle as the set of all points in a plane that are the same distance from the center.

7) Extend the idea to the set of all points in space that are the same distance from a center point. Contrast a circle and a sphere.